METHOD OF DESIGNING EXPLOSIVE-DRIVEN MAGNETIC FIELD GENERATORS

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A method of designing explosive-driven magnetic field generators that allows us to establish a dependence between the parameters of the generator circuit, in which the greatest energy release occurs under a time-invariant resistive load, is described. The problem of switching two-dimensional generators to a load whose resistance linearly increases with temperature is analytically solved as an example. The theoretical possibility of designing a generator in which the power released under the resistive load R(t) varies in a specified way with time is demonstrated. Types of current pulse, power, and energy released in the load are studied in the case of different generator circuit parameters.

§ 1. Explosive-driven magnetic field generators operating under the principle of rapid compression of the magnetic flux by means of an explosion, are the most powerful sources of pulsed currents [1-4]. The greatest attention in studies on explosive-driven magnetic field generators has been paid to their energy characteristics. Energy problems are related either to the obtaining of maximal energy in the active mode [5] or to attaining the greatest transfer of the energy of the explosive into electromagnetic energy (increase of generator efficiency) [6]. An analytic solution of such problems can be obtained only for a constant load R_0, such problems being numerically solved on a computer for an arbitrarily variable resistance. The use of explosive-driven magnetic field generators in plasma experiments [7] to obtain high magnetic pressures in isentropic compression of matter [8] or for other physical experiments related to problems in energetics raises problems associated with matching the generators to different loads and obtaining current pulses, power, and magnetic pressures that vary with time according to a given law.

The operation of an explosive-driven magnetic field generator on a concentrated active load will be considered within the framework of an electrical engineering model, according to which an explosive-driven magnetic field generator is represented as a decreasing inductance $L_1(t)$ connected to a resistance R(t) and inductance L_2 (Fig. 1). A current I_0 passes through the generator at the start of compression of the magnetic flux (t = 0). It is assumed in the calculations that losses of magnetic flux in the generator itself are slightly less than load losses.

Based on the induction law it can be found that current in the generator is described by the equation

$$d\ln I=(1-\rho)d\ln L,\ \ L=L_1(t)+L_2.$$

Current depends, in addition to inductance L, on the dimensionless function ρ (dimensionless circuit resistance) which is defined both as a resistance R(t) and (in terms of dL/dt) as the structural features of the explosive-driven magnetic field generator,

$$\rho(t)=-\frac{R(t)}{dL/dt}. \tag{1.1}$$

In some cases, the current equation can be obtained explicitly if the behavior of ρ(t) is specified. We may obtain different time dependences ρ(t) for the same value of R(t) by varying dL/dt, and succeed in transmitting different types of current in the load.

The dependence of generator current on the dimensionless combination of the cirucit parameters in Eq. (1.1) can be used to solve a number of problems in which the variation of R(t) has been preassigned, based on the experimental conditions (this involves the determination of a generator inductance optimal for using the ex-

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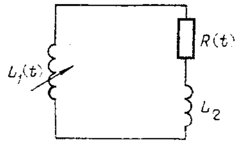

Fig. 1

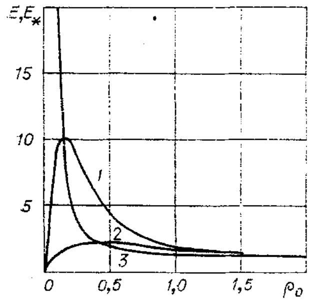

Fig. 2

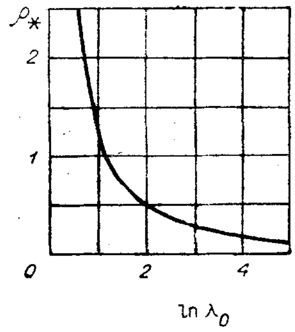

Fig. 3

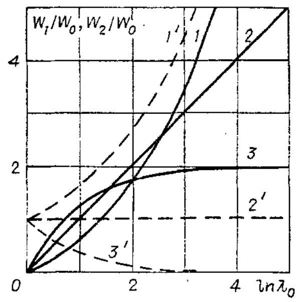

Fig. 4

plosive energy and the inductance at which the current pulse, power, and energy in the load vary with time in a specified way). We integrate Eq. (1.1), taking into account the actual conditions of the problem and the behavior of R(t) in order to solve them. Here ρ(t) is set equal to any function convenient for the calculation and that ensures the problem will be solved.

Explosive-driven magnetic field generators are the only energy source able to ensure the solution of these problems for arbitrary active loads.

The parameter ρ whose physical meaning is the ratio of compression time τ_0 to time of relaxation of the magnetic flux, $\tau_1 = L/R$, is sometimes interpreted as the inverse Reynolds magnetic number [7].

§ 2. Let us use our method to find the optimal (in terms of the energy of the explosive) inductance of an explosive-driven magnetic field generator operating on a heat-varying resistive load. The optimal (in terms of the energy of the explosive) generator will be that generator in which the greatest fraction of the energy of the explosive is transformed into electromagnetic energy. An optimality condition has been previously formulated [6] and holds if the power developed as the current-carrying circuit is deformed, is equal in any generator cross section to the maximal fraction of power developed as the explosive is exploded, i.e.,

$$-\frac{I^2}{2}\,dL/dt = kqS(x)\,D, \tag{2.1}$$

where q is the explosive energy per unit of volume, S(x) is the cross-sectional area of the explosive charge, x(t) is the current coordinate of the detonation front, D is the rate of detonation of the explosive, and k is the conversion ratio between explosive energy and electromagnetic energy.

If resistance linearly varies with increasing temperature $[R = R_0(1 + \alpha T)]$ as joule heat is released in it, we obtain

$$\frac{dR}{dt} = \frac{R_0\alpha}{C} RI^2, \quad (2.2)$$

where α is the temperature-resistance coefficient, C is the total heat capacity of the load, and R_0 is the initial resistance.

Equations (1.1), (2.1), and (2.2) determine a dependence of resistance on time,

$$R(t) = R_0\left(1 + \frac{2\alpha kqD}{C}\int_0^t S(\xi)\rho(\xi)\,d\xi\right)$$

and the inductance of an optimal generator

$$L_1(t) = L_1(0) - \int_0^t \frac{R_0}{\rho(\tau)}\left(1 + \frac{2\alpha kqD}{C}\int_0^\tau S(\xi)\rho(\xi)\,d\xi\right)d\tau.$$

§ 3. Let us find the optimal (in terms of the energy of the explosive charge) width of the busbars, current, power, and behavior of the resistive load with time as a two-dimensional explosive-driven magnetic field generator is connected to a resistance that linearly increases with temperature. The inductance of a two-dimensional generator is determined by the equation

$$L_1(t) = L_1(0) - \int_{-l}^{-l+Dt} \frac{4\pi b}{y(x)}\,dx. \quad (3.1)$$

Here it is assumed that the magnetic field in the generator is homogeneous, i.e., the distance between the busbars (2b = const) is less than the busbar width 2y(x), which varies throughout the length of the generator x(t). The busbars are closed at a rate equal to the detonation rate D; the total generator length is denoted by l and the coordinate origin is situated at the point at which the busbars are connected to the load. The cross section of an explosive charge with constant thickness 2δ is given by

$$S(x) = 4\delta y(x).$$

Equations (2.1) and (3.1) imply that the behavior of the current in an optimal two-dimensional generator coincides with the variation in the width of the busbars compressing the magnetic field,

$$I = I_0 y/y_0;\ y_0 = y(-l).$$

This is explained by the fact that the reserve of kinetic energy of any element of the conductor that it can obtain from an explosive charge with constant thickness remains invariant throughout the length of the generator if an explosive-driven magnetic field generator is optimized relative to k = const. The factor k will be optimal if the force acting on the conductor in the direction of the magnetic field remains constant along the explosive charge or along the stopping distance 2b = const. That is, the linear current density on the compression front of the magnetic field must be identical in a generator with variable busbar width.

We find the optimal busbar layout

$$y(x) = \frac{y_0 \exp\left(\beta\frac{x+l}{D}\right)}{\sqrt{1 + \frac{B}{2\beta}\left[\exp\left(2\beta\frac{x+l}{D}\right) - 1\right]}}, \quad B = \frac{16\alpha kq\delta y_0 D\rho_0}{C}$$

and increase in load resistance

$$R(t) = R_0\sqrt{1 + \frac{B}{2\beta}\left[\exp(2\beta t) - 1\right]}.$$

from the equations presented above, given $\rho = \rho_0\exp(\beta t)$, where β = const. We may determine the power $P = RI^2$ released in the load for known R(t) and I(t).

The nature of the variation of busbar width, current, resistance, and power depends to a significant degree on the parameter β in sufficiently long generators. When $\beta > 0$ and $t \to \infty$, busbar width and current approach the limits

$$y(x) = y_0\sqrt{2\beta/B},\ I = I_0\sqrt{2\beta/B},$$

while resistance and power exponentially increase,

$$R \to R_0 \sqrt{B/2\beta} \exp(\beta t), \; P \to R_0 I_0^2 \sqrt{2\beta/B} \exp(\beta t).$$

If $\beta < 0$, the resistance will be at most $R_0 \sqrt{1 - B/2\beta}$, while the busbar width, current, and power decrease to zero. When the active load is constant ($\alpha = 0$),

$$y(x) = y_0 \exp\left(\beta \frac{x+l}{D}\right),$$

which corresponds to a previously obtained [6] result.

The result for the operation of a two-dimensional generator with constant $\rho = \rho_0$ is qualitatively different. In this case, busbar width (and current) decreases,

$$y(x) = \frac{y_0}{\sqrt{1 + B\frac{x+l}{D}}},$$

while the resistive load increases according to a different time law,

$$R(t) = R_0 \sqrt{1 + Bt}.$$

A decrease in busbar width is entirely natural, since the condition $\rho =$ const holds only when $-dL/dt = 4\pi bD/y(x)$ increases with increasing resistance. Therefore, the requirement that $\rho =$ const, leads to a contraction of the generator busbars. The energy

$$W_1 = \int_0^{l/D} RI^2 dt = \frac{C}{\alpha}\left(\sqrt{1 + B\frac{l}{D}} - 1\right)$$

is released when the busbars are laid out in this way as the flux in the load is compressed, while the energy of the explosive charge used is given by

$$Q = \int_{-l}^{0} 4q\delta y(x)\, dx = \frac{C}{2\alpha k \rho_0}\left(\sqrt{1 + B\frac{l}{D}} - 1\right).$$

The ratio

$$\frac{W_1}{Q} = 2k\rho_0 \tag{3.2}$$

makes it clear that when $\rho_0 > 1/2$, energy that can be released from the resistance is greater than that contained in the explosive charge. This apparent contradiction can be explained in the following manner.

A power $-(I^2/2)(dL/dt)$ is released in an active load and is used to increase the energy of the magnetic field, while the energy equation has the form

$$\frac{-I^2}{2}\frac{dL}{dt} = RI^2 + \frac{d}{dt}\left(\frac{LI^2}{2}\right). \tag{3.3}$$

It is therefore evident that magnetic field energy is released from the resistance (in addition to the energy of the explosive charge) when $R > -1/2\,(dL/dt)$, so that Eq. (3.2) is not without meaning.

We integrate Eq. (2.1) with known cross section S(x) of the explosive charge, determining the inductance

$$L_1(t) = L_1(0) - \int_0^t \frac{2kqS(\xi)R(\xi)}{P(\xi)}\, d\xi$$

of the explosive-driven magnetic field generator that can develop a power $P = RI^2$ from a resistance R(t), this power varying with time in a given way.

§ 4. Let us consider the optimization problem for an explosive-driven magnetic field generator with respect to the release of the greatest energy for a variable active load.

We may assume that different amounts of energy will be released in the same actively loaded unit depending on R(t). When $R \to 0$, $W_1 \to 0$, while when $R \to \infty$ (due to rapid flux loss), energy somewhat exceeding the initial energy of the magnetic field ($W_0 = L_0 I_0^2/2$) cannot be expected. We have $W_1 = W_0 \ln(L_0/L_2)$ for

R_0 = const and $\rho = {}^1/_2$ [5]. Experiments have been described [7] on loading explosive-driven magnetic field generators to a special plasma load in such a way that the inverse Reynolds magnetic number was kept constant, and the energy in it was calculated.

It is of greater interest to find a dependence between the parameters of an explosive-driven magnetic field generator under which the greatest energy is released as it operates with an arbitrarily varying resistive load. This problem can be completely solved when $\rho = \rho_0$ = const.

We then find that the current is given by

$$I = I_0 \lambda^{1-\rho_0};$$

the resistive energy by

$$W_1 = \int R I^2 dt = W_0 \frac{2\rho_0}{2\rho_0 - 1} (1 - \lambda^{1-2\rho_0}); \tag{4.1}$$

and the magnetic field energy of the generator by

$$W_2 = LI^2/2 = W_0 \lambda^{1-2\rho_0}$$

where $\lambda = L_0/L(t)$, $\lambda_0 = L_0/L_2$ are the current and total adjustment factors of the generator and $L_0 = L_1(0) + L_2$ is the initial inductance of the circuit.

Figure 2 depicts $E = W_1/W_0$ as a function of ρ when $\lambda_0 = 100$ and 10 (curves 1 and 2), so that it is evident that there exists a unique ρ_* for a given λ_0 at which energy has maximal value E_* [$E_*(\rho_*, \lambda_0)$ is depicted by curve 3]. The value of ρ_* is determined by solving the transcendental equation

$$\frac{dE}{2d\rho_0} = -\frac{1}{(2\rho_0 - 1)^2} + \frac{\lambda_0^{1-2\rho_0}}{(2\rho_0 - 1)^2} + \frac{2\rho_0}{2\rho_0 - 1} \lambda_0^{1-2\rho_0} \ln \lambda_0 = 0. \tag{4.2}$$

We introduce the variable

$$\xi = (1 - 2\rho_*) \ln \lambda_0,$$

making it possible to reduce Eq. (4.2) (to determine the optimal ρ_* and λ_0) to the system of equations

$$\begin{cases} 2\rho_* = 1 - \xi/\ln \lambda_0; \\ 2\rho_* = \dfrac{1 - \exp(-\xi)}{\xi}, \end{cases} \tag{4.3}$$

where λ_0 is a parameter. We find $\rho_*(\lambda_0)$ (Fig. 3) by graphically determining λ_0 from Eqs. (4.3).

An analysis of E_* and $W_* = W_2/W_0$ shows that significant release of energy on the active load may occur at low ρ_*. When $\rho_* \gg 1$, the resistive energy is close to the initial energy of the magnetic field.

The chief part of the eenrgy is released due to resistance in generators with $\rho_0 = \rho_*$ and low adjustment factor $\lambda_0 < e^2$. When $\lambda_0 = e^2$ the energy released in a resistive load at the end of generator operation is twice the energy stored in the magnetic field. The energy stored in the magnetic field exceeds the energy due to resistance in generators with a high adjustment factor ($\lambda_0 > e^2$).

Equation (4.1) implies that the energy released due to resistance does not have a maximum as a function of λ_0 for a given ρ_0. When $\rho_0 < 1/2$ energy W_1 monotonically increases together with λ_0 up to $W_1 = W_0 2\rho_0 \lambda^{1-2\rho_0}/(1-2\rho_0)$. When $\rho_0 < 1/2$, the energy in the resistive load increases with increasing λ_0, remaining bounded by $W_1 \approx W_0[2\rho_0/(2\rho_0 - 1)]$. Let us consider the operating conditions of an explosive-driven magnetic field generator when $\rho_0 = 1/2$. It is clear from the energy equation (3.3) that the energy of the magnetic field in the generator remains constant when $R(t) = -{}^1/_2 dL/dt$ and that all the power developed as the current-carrying circuit is deformed by applied forces is consumed in heating the conductors, i.e., $W_2 = W_0$ and $W_1 = W_0 \ln \lambda_0$.

In Fig. 2 the vertical line passing through $\rho_0 = 0.5$ corresponds to this E. The energy release due to resistance under these operating conditions is less than for optimal $\rho_* \neq {}^1/_2$. Figure 4 depicts resistive energy release (unbroken curves) and the magnetic field energy (broken curves) for a given ρ_0 as a function of λ_0, where $\rho_0 = 0.25$ for curves 1 and 1', $\rho_0 = 0.5$ for curves 2 and 2', and $\rho_0 = 1$ for curves 3 and 3'.

Thus, a calculation of the parameters of an explosive-driven magnetic field generator in which energy W_1 is released for resistance R(t) in the course of operation, reduces to the following. We specify the ratio W_1/W_0, taking into account the initial energy W_0, and find the optimal ρ_* from curve 3 in Fig. 2

to which λ_0 corresponds in Fig. 3. We determine the desired variation of the generator inductance from the value of ρ_* found and from the given R(t) by integrating Eq. (1.1).

A study of the operation of explosive-driven magnetic field generators demonstrates that the selection of a particular dependence between its parameters (explosive charge stopping distance, load resistance, etc.) allows us to:

1) determine the optimal (relative to the use of the energy of the explosive) inductance of an explosive-driven magnetic field generator connected to a resistance that varies with temperature;

2) construct generators that can develop power varying with time according to a given law for a resistance R(t);

3) find an inductance for arbitrary load R(t), setting ρ = const, such that the greatest amount of energy is released for resistance during the operation of the explosive-driven magnetic field generator.

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